Electromagnetic Wave Propagation انتشار الموجات الكهرومغناطيسية

Our first application of Maxwell's equations will be in relation to electromagnetic wave (EM-Wave) propagation. The existence of EM waves, predicted by Maxwell's equations, was first investigated by Heinrich Hertz. After several calculations and experiments Hertz succeeded in generating and detecting radio waves

In general, waves are means of transporting energy or information

In this chapter, our major goal is to solve Maxwell's equations and derive EM wave motion in the following media:

$1 -$ $\boldsymbol{Free\ space}$ $(\sigma = 0 \ \ , \varepsilon = \varepsilon_0 \ \ , \mu = \mu_0)$

$2 -$ $\boldsymbol{Lossless\ dielectrics}$ $(\sigma = 0 \ \ , \varepsilon = \varepsilon_r \varepsilon_0 \ \ , \mu = \mu_r \mu_0 \ \ , \sigma \ll \omega\varepsilon)$

$3 -$ $\boldsymbol{Lossy\ dielectrics}$ $(\sigma \neq 0 \ \ , \varepsilon = \varepsilon_r \varepsilon_0 \ \ , \mu = \mu_r \mu_0$

$2 -$ $\boldsymbol{Good\ conductors}$ $(\sigma \cong \infty \ \ , \varepsilon = \varepsilon_0 \ \ , \mu = \mu_r \mu_0 \ \ , \sigma \gg \omega\varepsilon)$

Before we consider wave motion in those different media, it is appropriate that we study the characteristics of waves in general. This is important for proper understanding of **EM *waves***

10.1 <u>Waves in General</u>

With time dependence $e^{j\omega t}$ for both E and H, Maxwell's equations become

$\nabla \times E = -j\omega\mu\, H$

$\nabla \times H = (\sigma + j\omega\mu)\, E$

$\nabla . \mathrm{E} = 0$

$\nabla . \mathrm{H} = 0$

Taking the curl

$\nabla \times (\nabla \times E) = -j\omega\mu\, (\nabla \times \ H)$

$\nabla \times (\nabla \times H) = (\sigma + j\omega\mu)\, (\nabla \times E)$

$\nabla^2 H = \gamma^2 H$

$\nabla^2 E = \gamma^2 E$

γ is the propagation constant

$\gamma = \alpha + j\beta$

Of particular interest are solutions (plane waves) that depend on only one spatial coordinate, say z. Then the equation becomes

$$\frac{d^2 H}{dz^2} = \gamma^2 H$$

which, for an assumed time dependence $e^{j\omega t}$, is the vector analog of the one-dimensional scalar wave equation. Solutions are as above, in terms of the propagation constant γ :

$$H(z,t) = H_0 e^{\mp\gamma z}\, e^{j\omega t}\, \mathrm{a}_H$$

$$E(z,t) = E_0 e^{\mp\gamma z}\, e^{j\omega t}\, \mathrm{a}_E$$

Or

$$E = E_0\, e^{-\alpha z} \cos(\omega t - \beta z)$$

$$\beta = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

Note the following characteristics of the wave in eq. above

1- It is time harmonic because we assumed time dependence $e^{j\omega t}$

2- E_0 is called the *amplitude* of the wave and has the same units as *E*.

3- $(\omega t - \beta z)$ is the *phase* (in radians) of the wave; it depends on time *t* and space variable z

4- ω is the *angular frequency* (in radians/second); β is the *phase constant* or *wave number* (in radians/meter).

Example: The electric field is given by:

$$E = 50\cos(10^8 t + \beta x)\, \mathrm{a}_y \quad V/m$$

(a) Find the direction of wave propagation.

(b) Calculate $\beta and\ \omega$

Solution:

(a) From the positive sign in $(10^8 t + \beta x)$, we infer that the wave is propagating along $-\mathrm{a}_x$

(b) $\omega = 10^8$

$$\beta = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = 1/3 \quad rad/m$$

10.2 Wave Propagation in Lossy Dielectrics

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A ***lossy dielectric*** is a medium in which an EM wave loses power as it propagates due to poor conduction

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

Where α attenuation constant or *attenuation factor*

$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \mathrm{a}_x \quad V/m$$

The ***intrinsic impedance*** η (in ohms) of the medium is given by:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\mu}} = |\eta| \angle \theta_\eta$$

$$\eta = \frac{E}{H}$$

$$H = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \mathrm{a}_y \quad A/m$$

$$\tan(2\theta_\eta) = \frac{\sigma}{\omega\varepsilon} \qquad \text{"}Loss\ tangent\text{"}$$

Notice from equations above that as the wave propagates along a_z, *it decreases or attenuates in amplitude by a factor* $e^{-\alpha z}$

10.3 Wave Propagation in Lossless Dielectrics

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In the lossless dielectric, $\sigma \ll \omega\varepsilon$. it is a special case of that in section 10.2

$$\sigma = 0 \quad , \varepsilon = \varepsilon_r \varepsilon_0 \quad , \mu = \mu_r \mu_0$$

Substituting gives:

$$\alpha = 0 \qquad , \beta = \omega\sqrt{\mu\varepsilon}$$

Also

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \lfloor 0$$

And thus E and H are in time phase with each other

$$E(z,t) = E_0 \cos(\omega t - \beta z)\, \mathrm{a}_x \quad V/m$$

$$H = \frac{E_0}{|\eta|} \cos(\omega t - \beta z)\, \mathrm{a}_y \quad A/m$$

10.4 Wave Propagation in Free Space

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This is a special case of what we considered in Section 10.2. In this case, *a*

$$\sigma = 0 \quad , \varepsilon = \varepsilon_0 \quad , \mu = \mu_0 \qquad , \qquad \varepsilon_r = \mu_r = 1$$

Substituting gives:

$$\alpha = 0 \qquad , \beta = \omega\sqrt{\mu_0 \varepsilon_0} = \frac{\omega}{c}$$

Where the speed of light in a vacuum $c = 3 \times 10^8 \; m/s$

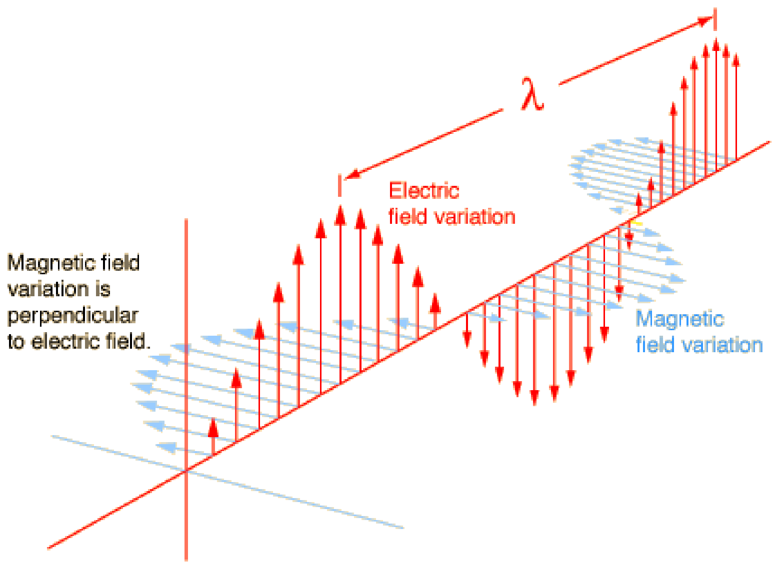
Also

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \;\; \Omega \; = 377\Omega$$

$$E(z,t) = E_0 \cos(\omega t - \beta z)\, \mathrm{a}_x \quad V/m$$

$$H = \frac{E_0}{120\pi} \cos(\omega t - \beta z)\, \mathrm{a}_y \quad A/m$$

The plots of E and H are shown in Figure below:



Both E and H fields (EM waves) are everywhere ***normal to the direction of wave propagation***, *ak*. That means, the fields lie in a plane that is transverse or orthogonal to the direction of wave propagation. **They form an EM wave that has no electric or magnetic field components along the direction of propagation; such a wave is called a *transverse electromagnetic* (TEM) wave**.

10.5 Wave Propagation in Good Conductor

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A perfect, or good conductor, is one in which $\sigma \gg \omega\varepsilon$ we so that $\frac{\sigma}{\omega\varepsilon} \cong \infty$; that is

$$\sigma \cong \infty \quad ,\varepsilon = \varepsilon_0 \quad ,\mu = \mu_r\mu_0$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\ \lfloor 45^0$$

and thus E leads H by 45°. If

$$E(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\,\mathrm{a}_x \quad V/m$$

$$H = \frac{E_0}{|\eta|} e^{-\alpha z}\cos\left(\omega t - \beta z - 45^0\right)\mathrm{a}_y \quad A/m$$

Therefore, as E (or H) wave travels in a conducting medium, its amplitude is attenuated by the factor $e^{-\alpha z}$. The distance δ through which the wave amplitude decreases by a factor e^{-1} (about 37%) is called ***skin depth*** **or** ***penetration depth*** of the medium; that is,

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

The **skin depth** is a measure of the depth to which an EM wave can penetrate the medium.

Example: A uniform plane wave propagating in a medium has

$\boldsymbol{E = 2e^{-\alpha z} \sin(10^8 t - \beta z)\, a_y}\ \ V/m$ .If the medium is characterized by $\varepsilon_r = 1$, $\mu_r = 20$ and $\sigma = 3$ mhos/m, find α , β and **H.**

Solution:

We need to determine the loss tangent to be able to tell whether the medium is a lossy dielectric or a good conductor

$$\frac{\sigma}{\omega\varepsilon} = \frac{3}{10^8 \times 1 \times 8.85 \times 10^{-12}} = 3393 \gg 1$$

Showing that the medium may be regarded as a ***good conductor*** at the frequency of operation. Hence,

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{10^8 \times 20 \times 4\pi \times 10^{-7} \times 3}{2}} = 61.4\ \ Np/m$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\ \lfloor 45^0 = \sqrt{\frac{10^8 \times 20 \times 4\pi \times 10^{-7}}{3}} \lfloor 45^0 = \sqrt{\frac{800\pi}{3}} \lfloor 45^0$$

$$H = \frac{2}{\sqrt{\frac{800\pi}{3}}} e^{-\alpha z} \sin(\mathbf{10^8} t - \beta z - 45^0)\, \mathrm{a}_H \ \ A/m$$

$$\mathrm{a}_H = \mathrm{a}_k \times \mathrm{a}_E = \mathrm{a}_z \times \mathrm{a}_y = -\mathrm{a}_x$$

$$H = -69.1\ e^{-61.42 z} \sin(10^8 t - 61.42\, z - 45^0)\, \mathrm{a}_x \ \ A/m$$

Example: In a homogeneous nonconducting region where μ_r = 1, find ε_r and ω if

$$E = 30\pi e^{j\left(\omega t - \frac{4}{3}y\right)}\mathrm{a}_z \qquad H = e^{j\left(\omega t - \frac{4}{3}y\right)}\mathrm{a}_x?$$

Solution:

Compared with:

$$E = E_0 e^{-\alpha z} e^{j(\omega t - \beta y)}\mathrm{a}_z$$

Then:

$$\alpha = 0 \qquad ,\beta = \frac{4}{3}$$

The media is a ***lossless dielectric*** because $\alpha = 0$ and $E/H \neq 120\pi$

$$\eta = \frac{|E|}{|H|} = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\frac{30\pi}{1} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}} = 120\pi\sqrt{\frac{1}{\varepsilon_r}}$$

$$\sqrt{\frac{1}{\varepsilon_r}} = \frac{30\pi}{120\pi}$$

$$\varepsilon_r = 16$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$\frac{4}{3} = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$$

$$\sqrt{\mu_0\varepsilon_0} = \frac{1}{c} = \frac{1}{3\times 10^8}$$

$$\frac{4}{3} = \frac{1}{3\times 10^8}\omega\sqrt{\mu_r\varepsilon_r}$$

$$\frac{4}{3} = \frac{\omega}{3\times 10^8}\sqrt{1\times 16}$$

$$\omega = \frac{4\times 3\times 10^8}{3\times 4} = 10^8\ rad/sec$$

Example: Determine the propagation constant γ for a material having $\mu_r = 1,\ \varepsilon_r = 8$, $\sigma = 0.25\ PS/m$ if the wave frequency is 1.6 MHz?

Solution:

$$\frac{\sigma}{\omega\varepsilon} = \frac{0.25 \times 10^{-12}}{(2\pi \times 1.6 \times 10^{6}) \times 8 \times 8.85 \times 10^{-12}} = 10^{-9} \ll 1$$

So that the material is lossless material

$$\alpha = 0$$

$$\beta = \omega\sqrt{\mu\varepsilon} = 2\pi f\,\frac{\sqrt{\mu_r\varepsilon_r}}{c} = 2\pi \times 1.6 \times 10^{6}\,\frac{\sqrt{1 \times 8}}{3 \times 10^{8}} = 9.48 \times 10^{-2}\ rsd/m$$

$$\gamma = \alpha + j\beta = 0 + j9.48 \times 10^{-2} = j9.48 \times 10^{-2}$$

Example: Find the skin depth δ at a frequency of 1.6 MHz in aluminum, where σ = 38.2 MS/m and μ_r = 1. Also find γ?

Solution:

$$\delta = \frac{1}{\sqrt{\pi f\mu\sigma}} = \frac{1}{\sqrt{\pi \times 1.6 \times 10^{6} \times 1 \times 4\pi \times 10^{-7} \times 38.2 \times 10^{6}}} = 64.6\ \mu m$$

$$\alpha = \beta = \frac{1}{\delta} = \frac{1}{64.6 \times 10^{-6}} = 1.55 \times 10^{4}$$

$$\gamma = \alpha + j\beta = 1.55 \times 10^{4} + j1.55 \times 10^{4}$$

Example: In free space $\boldsymbol{E} = \mathbf{10^3\,sin}(\boldsymbol{\omega t} - \boldsymbol{\beta z})\,\mathbf{a}_{\boldsymbol{y}}\ \ V/m$ (V/m). Obtain H(z, t).?

Solution:

In free space

$$\frac{E}{H} = \eta = 120\pi$$

$$\mathrm{a}_H = \mathrm{a}_k \times \mathrm{a}_E = \mathrm{a}_z \times \mathrm{a}_y = -\mathrm{a}_x$$

$$H = \frac{E}{120\pi} = \frac{\mathbf{-10^3\,sin}(\boldsymbol{\omega t} - \boldsymbol{\beta z})\mathrm{a}_x}{120\pi}$$

Appendix

$$\nabla . D = \frac{\partial D_x}{\partial x}\mathrm{a}_x + \frac{\partial D_y}{\partial y}\mathrm{a}_y + \frac{\partial D_z}{\partial z}\mathrm{a}_z \qquad \text{(Cartesian)}$$

$$\nabla . \mathrm{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho D_\rho + \frac{1}{\rho}\frac{\partial D_\emptyset}{\partial \emptyset} + \frac{\partial D_z}{\partial z} \qquad \text{(cylindrical)}$$

$$\nabla . \mathrm{D} = \frac{1}{r^2}\frac{\partial}{\partial r}r^2 D_r + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\emptyset}{\partial \emptyset} \qquad \text{(spherical)}$$

$$\nabla V = \frac{\partial V}{\partial x}\mathrm{a}_x + \frac{\partial V}{\partial y}\mathrm{a}_y + \frac{\partial V}{\partial z}\mathrm{a}_z \qquad \text{(Cartesian)}$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \emptyset}\mathbf{a}_\emptyset + \frac{\partial V}{\partial z}\ \mathbf{a}_z \qquad \text{(cylindrical)}$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \emptyset}\mathbf{a}_\emptyset \qquad \text{(spherical)}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \qquad \text{(Cartesian)}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \emptyset^2}\right) + \frac{\partial^2 V}{\partial z^2} \qquad \text{(cylindrical)}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \emptyset^2} \qquad \text{(spherical)}$$

$$\nabla \times \mathrm{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_{x} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_{\mathbf{y}} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_{z} \qquad \text{(Cartesian)}$$

$$\nabla \times \mathrm{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \emptyset} - \frac{\partial H_\emptyset}{\partial z}\right)\mathbf{a}_{\rho} + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\mathbf{a}_\emptyset + \left(\frac{1}{\rho}\frac{\partial(\rho H_\emptyset)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \emptyset}\right)\mathbf{a}_{z} \qquad \text{(cylindrical)}$$

$$\nabla \times \mathrm{H} = \frac{1}{r\,sin\,\theta}\left(\frac{\partial(\sin\theta\,\mathrm{H}_\emptyset)}{\partial \theta} - \frac{\partial H_\theta}{\partial \emptyset}\right)\mathbf{a}_{r} + \frac{1}{r}\left(\frac{1}{sin\,\theta}\frac{\partial H_r}{\partial \emptyset} - \frac{\partial(rH_\emptyset)}{\partial r}\right)\mathbf{a}_{\theta} + \frac{1}{r}\left(\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right)\mathbf{a}_\emptyset \qquad \textit{(spherical)}$$

$\varepsilon_0 = \frac{1}{36\pi}10^{-9} = 8.854 \times 10^{-12}$ F/m $\qquad$ $\mu_o = 4\pi \times 10^{-7}$ H/m